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INFRARED SINGULARITIES OF FERMION PROPAGATOR AND THEIR CONNECTION WITH THE WILSON LOOP

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The factorization of infrared singularities of gauge-invariant spinor propagator is proved in the framework of QED. It turns out that this infrared factor coincides with the Wilson loop and accumulates all the dependence on the form of the path of the initial Green function.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Инфракрасные особенности фермионного пропагатора и их связь с петлей Вильсона

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Показана факторизация инфракрасных особенностей калибровочно-инвариантного спинорного пропагатора в КЭД. Установлено, что при этом инфракрасный множитель совпадает с петлей Вильсона и аккумулирует всю зависимость от контура исходной функции Грина.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The present work is devoted to the study of infrared asymptotics of a gauge-invariant (GI) spinor Green function. The interest in this problem comes from the hope that the problem of quark confinement in the framework of QCD can be solved in this way (see, for instance /1-4/, and the references, therein). Usually the standard fermion propagator $<0|T\psi(x)\bar{\psi}(y)|0>$ is studied that, as it is well-known /5/, is a gauge-dependent quantity. Thus, as it is shown, for example, in /6/, the infrared asymptotics of such a propagator is defined by the vacuum average of the path-exponential along the unclosed path.

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At the same time it is known that the infrared behaviour of the complete fermion propagator essentially depends on the gauge choice. Thus, for example, in the Abelian case in the class of covariant α-gauges the fermion propagator has a branch point at $p^2 = m^2$, which only at $\alpha = 3$ (Soloviev-Yennie gauge) leads to a pole singularity of the propagator. That is why one can conclude that a consistent study of the gauge-theories should be done on the basis of the gauge-independent quantities. In particular, instead of the standard spinor propagator one can consider a gauge-invariant Green function*

$$G^{GI}(x,y|C) = -\langle 0|T\psi(x)\mathcal{P}\exp\{-ie\int_x^y dz^\mu A_\mu(z)\}\bar{\psi}(y)\,|\,0\rangle. \qquad (1)$$

In contradistinction with the standard propagator the Green function (1) contains the exponential with the path integral taken over the gauge field along an arbitrary path C that connects the points x and y. Our aim consists in studing the infrared behaviour of the path-dependent gauge-invariant propagator (1).

Let us use the representation (1) in a form of the functional integral over the spinor and vector fields

$$G(x,y|C) =$$

$$= -\int D[\psi,\bar{\psi}]\cdot DA\cdot\psi(x)\mathcal{P}\exp\{-ie\int_x^y dz^\mu A_\mu(z)\}\bar{\psi}(y)\,. \qquad (2)$$

Here the integration measure over the gauge field DA includes some gauge condition

$$DA = \bar{D}A\cdot\delta[f(A)] \qquad (3)$$

the explicit form of which due to the gauge independence of (1) is not essential. Performing in (2) the integration over the fermionic fields we get (with $f(A) = \partial^\mu A_\mu(x)$)

$$G(x,y|C) = \int\bar{D}A\cdot\delta(\partial^\mu A_\mu)\cdot\frac{\det[\gamma^\mu D_\mu - m]}{\det[\gamma^\mu\partial_\mu - m]}\times$$
$$\times\exp[-S_0(A)]\cdot G(x,y|A)\cdot\exp[-ie\int_x^y dz^\mu A_\mu(z)]\,, \qquad (4)$$

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* We consider QED in Euclidean space-time.

where $S_0[A]$ is the Euclidean action of the free electromagnetic field $S_0[A] = \frac{1}{4}\int d^4x\, F_{\mu\nu}(x)\, F^{\mu\nu}(x)$; $D_\mu = \partial_\mu + ieA_\mu$, i.e., is a covariant derivative, and $G(x, y|A)$ is the Green function of the electron in an external field A_μ , which satisfies the equation

$$[\hat{D}_x - m]\, G(x, y|A) = \delta(x-y) .$$

In what follows we shall represent the field A_μ in accordance with /7/ as a sum of slowly and rapidly varying components $A_\mu^{(1)}$ and $A_\mu^{(0)}$. Then in (5) we shall make use of the formula /7/

$$G(x, y\,|\,A^{(0)} + A^{(1)}) \simeq G(x, y|A^{(1)}) \cdot \exp[ie \int_x^y dz^\mu A_\mu^{(0)}(z)] , \qquad (5)$$

that is valid in the infrared limit. The integration in (5) is performed along the piece of the straight line Π that connects the points x and y:

$$\Pi: \quad z^\mu = x^\mu + s(y-x)^\mu , \quad 0 \le s \le 1 . \qquad (6)$$

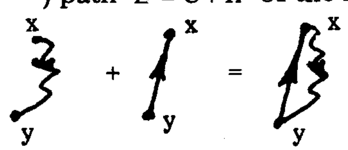
With account of formula (5) and the approximate ralation $\det[\hat{D} - m] \times \det^{-1}[\hat{\partial} - m] \simeq 1$, that holds for the infrared limit, it is not difficult to show that the propagator (4) can be represented as a product of two factors

$$G^{GI}(x, y|C) = J^{UV} \cdot J^{IR} . \qquad (7)$$

Here the first factor J^{UV} (UV-ultraviolet) is the quantum Green function obtained only with account of the rapidly varying field $A_\mu^{(1)}$. The second factor J^{IR} (IR — infrared) is obtained with account of slowly varying component $A_\mu^{(C)}$ only. It reflects the interaction with the soft photons and thus contains the infrared singularities. This factor has the form

$$J^{IR} = \int DA^{(0)} \cdot \exp[-ie \oint_L dz^\mu A_\mu^{(0)}(z)] , \qquad (8)$$

where $\oint_L dz^\mu A_\mu^{(0)}(z)$ is the integral over the closed (in the contradistinction with /6/) path $L = C + \Pi$ of the form

$L = C + \Pi =$  .

It is not difficult to see that the expression (11) is nothing but the Wilson loop $J^{IR} = W(L)$, where

$$W(L) = <0|T \exp\{-ie \oint_L dz^\mu \cdot A_\mu^{(0)}(z)\}|0> . \tag{9}$$

Thus, we have arrived to an interesting conclusion that all the properties of the infrared behaviour of the gauge-invariant propagator (1) are accumulated in the Wilson loop (9).

Let us consider, as an example, a particular choice of the path C as the piece of the straight line from the point x up to the point y*. With such a choice of the path the propagator (1) takes the form

$$G^{GI}(x, y|C=\Pi) = -<0|T\psi(x) \times$$

$$\times \exp[ie \int_0^1 d\alpha (y-x)^\nu A_\nu (x+\alpha(y-x))] \cdot \bar{\psi}(y)|0> , \tag{10}$$

where the integration in the exponential is performed along the piece of the straight line $C = \Pi_{xy}$ of form (6). It is clear that in this case

$$\oint_L dz^\mu A_\mu(z) = \int_x^y dz^\mu A_\mu(z) + \int_y^x dz^\mu \cdot A_\mu(z) = 0$$

so $J^{IR} = 1$. Thus, additional infrared singularities (like a branch point) that appear due to the interaction with the "soft" photons do not appear here, and the Fourier transform of the propagator $G^{GI}(x, y|C=\Pi)$ in the infrared limit has a simple pole $G^{GI}(p|C=\Pi) \sim \frac{1}{\hat{p}+m}$. This result exactly agrees with that of the calculation of the infrared asymptotics of the propagator (1) done in /8/.

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* With such a choice of the path C the propagator (1) coinsides with the standard propagator $<0|T\psi(x)\bar{\psi}(y)|0>$ if the fields A, ψ and $\bar{\psi}$ are taken in the gauge $(\tau - (\frac{x+y}{2}))\, A_\mu(\tau) = 0$, that is a particular case of the Fock gauge (see /8/).

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